# Edexcel Maths C4

Topic Questions from Papers

Differentiation

# 2. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where  $\frac{dy}{dx} = 0$ .

**(7)** 

Q2

(Total 7 marks)

1.	A curve	<i>C</i> is	described	by	the	equation
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$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point (1, -2), giving your answer in the form ax + by + c = 0, where a, b and c are integers.

1.	A curve	<i>C</i> is	described	by	the	equation
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$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form ax + by + c = 0, where a, b and c are integers.

(7)

**3.** A curve has parametric equations

$$x = 7\cos t - \cos 7t$$
,  $y = 7\sin t - \sin 7t$ ,  $\frac{\pi}{8} < t < \frac{\pi}{3}$ .

(a) Find an expression for  $\frac{dy}{dx}$  in terms of t. You need not simplify your answer.

**(3)** 

(b) Find an equation of the normal to the curve at the point where  $t = \frac{\pi}{6}$ .

Give your answer in its simplest exact form.

**(6)** 

- 5. A set of curves is given by the equation  $\sin x + \cos y = 0.5$ .
  - (a) Use implicit differentiation to find an expression for  $\frac{dy}{dx}$ .

**(2)** 

For  $-\pi < x < \pi$  and  $-\pi < y < \pi$ ,

(b) find the coordinates of the points where  $\frac{dy}{dx} = 0$ .

(5)

. (a)	Given that $y = 2^x$ , and using the result $2^x = e^{x \ln 2}$ , or otherwise, show that $\frac{dy}{dx} = 2^x \ln 2$ .
(b)	Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2,16).

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5.	A curve	is	described	by	the	equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where x = -8.

(3)

(b) Find the gradient of the curve at each of these points.

**(6)** 

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<b>4.</b> A curve has equation $3x^2 - y^2 + xy = 4$ . The points <i>P</i> and <i>Q</i> lie on the curve. of the tangent to the curve is $\frac{8}{3}$ at <i>P</i> and at <i>Q</i> .	The gradient
(a) Use implicit differentiation to show that $y - 2x = 0$ at $P$ and at $Q$ .	(6)
(b) Find the coordinates of $P$ and $Q$ .	(3)

- 1. A curve C has the equation  $y^2 3y = x^3 + 8$ .
  - (a) Find  $\frac{dy}{dx}$  in terms of x and y.

**(4)** 

(b) Hence find the gradient of C at the point where y = 3.

(3)

- **4.** The curve C has the equation  $ye^{-2x} = 2x + y^2$ .
  - (a) Find  $\frac{dy}{dx}$  in terms of x and y.

**(5)** 

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form ax + by + c = 0, where a, b and c are integers.

**(4)** 

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3. The curve C has the equation

$$\cos 2x + \cos 3y = 1$$
,  $-\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$ ,  $0 \leqslant y \leqslant \frac{\pi}{6}$ 

(a) Find  $\frac{dy}{dx}$  in terms of x and y.

**(3)** 

The point *P* lies on *C* where  $x = \frac{\pi}{6}$ .

(b) Find the value of y at P.

**(3)** 

(c) Find the equation of the tangent to C at P, giving your answer in the form  $ax + by + c\pi = 0$ , where a, b and c are integers.

**(3)** 

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$2^x + y^2 = 2xy$	
Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).	
	(7)

ln y = 2x ln x,  x > 0, y > 0	
at the point on the curve where $x = 2$ . Give your answer as an exact value.	(=)
	(7)

Leave

] ]	The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$ . The point P on the curve has coordinates $(-1, 1)$ .	
(	(a) Find the gradient of the curve at P.	(5)
(	(b) Hence find the equation of the normal to $C$ at $P$ , giving your answer in the $ax+by+c=0$ , where $a$ , $b$ and $c$ are integers.	form
		(3)

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find  $\frac{dy}{dx}$  in terms of x and y.

**(5)** 

(b) Find the coordinates of the points on C where  $\frac{dy}{dx} = 0$ .

2.	The curve	C has	equation
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$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that  $\frac{dy}{dx}$  at the point (1, 3) on the curve C can be written in the form  $\frac{1}{\lambda} \ln(\mu e^3)$ ,

where  $\lambda$  and  $\mu$  are integers to be found.

7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(a) Find  $\frac{dy}{dx}$  in terms of x and y.

**(5)** 

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x coordinate of Q is negative,

(b) use your answer to part (a) to find the coordinates of Q.



Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$$f(x) \qquad \int f(x) dx$$

$$\sec^2 kx \qquad \frac{1}{k} \tan kx$$

$$\tan x \qquad \ln|\sec x|$$

$$\cot x \qquad \ln|\sin x|$$

$$\csc x \qquad -\ln|\csc x + \cot x|, \quad \ln|\tan(\frac{1}{2}x)|$$

$$\sec x \qquad \ln|\sec x + \tan x|, \quad \ln|\tan(\frac{1}{2}x + \frac{1}{4}\pi)|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

### Logarithms and exponentials

$$e^{x \ln a} = a^x$$

### Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

## Differentiation

f(x) f'(x)  
tan kx 
$$k \sec^2 kx$$
  
sec x  $\sec x \tan x$   
cot x  $-\csc^2 x$   
cosec x  $-\csc x \cot x$   

$$\frac{f(x)}{g(x)} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^{n} = a^{n} + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^{2} + \dots + \binom{n}{r} a^{n-r}b^{r} + \dots + b^{n} \quad (n \in \mathbb{N})$$
where  $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ 

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{1 \times 2}x^{2} + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r}x^{r} + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{\infty} = \frac{a}{1-r}$$
 for  $|r| < 1$ 

Numerical integration

The trapezium rule: 
$$\int_{a}^{b} y \, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$$
, where  $h = \frac{b - a}{n}$ 

#### Mensuration

Surface area of sphere =  $4\pi r^2$ 

Area of curved surface of cone =  $\pi r \times \text{slant height}$ 

#### Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$$